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Spherical Dust Collapse in Higher Dimensions

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We consider here the question if it is possible to recover cosmic censorship when a transition is made to higher dimensional spacetimes, by studying the spherically symmetric dust collapse in an arbitrary higher spacetime dimension. It is pointed out that if only black holes are to result as end state of a continual gravitational collapse, several conditions must be imposed on the collapsing configuration, some of which may appear to be restrictive, and we need to study carefully if these can be suitably motivated physically in a realistic collapse scenario. It would appear that in a generic higher dimensional dust collapse, both black holes and naked singularities would develop as end states as indicated by the results here. The mathematical approach developed here generalizes and unifies the earlier available results on higher dimensional dust collapse as we point out. Further, the dependence of black hole or naked singularity end states as collapse outcomes, on the nature of the initial data from which the collapse develops, is brought out explicitly and in a transparent manner as we show here. Our method also allows us to consider here in some detail the genericity and stability aspects related to the occurrence of naked singularities in gravitational collapse.

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I. INTRODUCTION

A considerable debate has continued in recent years on the validity or otherwise of the cosmic censorship conjecture (CCC) in black hole physics (see e.g. [1-7] for some recent reviews). The reason for this interest is that CCC is fundamental to many aspects of theory and applications of black hole physics, and the astrophysical implications resulting from a continual gravitational collapse of a massive star which has exhausted its nuclear fuel. As of today, no theoretical proof or any satisfactory mathematical formulation of CCC is available, where as many collapse scenarios have been analyzed and worked out wherein the collapse end state is either a *black hole* (BH) or a *naked singularity* (NS), depending on the nature of the initial data from which the collapse evolves developing from a regular initial state to the final super-dense state. This has important astrophysical significance for the reason that naked singularities may have observational properties which could be radically different from those of a black hole.

A note-worthy suggestion that has emerged towards a possible theoretical formulation of CCC is that any naked singularities resulting from matter models which may also develop singularities in special relativity, should not be regarded as physical (see e.g. [3]). Clearly, it will require a serious effort to cast such a possibility into a mathematical statement and a proof for CCC. Also, it may not be easy to discard completely all the matter fields such as dust, perfect fluids, and matter with various other reasonable equations of state, which have been studied and used extensively in relativistic astrophysics for a long time.

Another possibility that indeed appears worth exploring is we may actually be living in a higher dimensional spacetime universe. The recent developments in string theory and other field theories strongly indicate that gravity is possibly a higher dimensional interaction, which reduces to the general relativistic description at lower energies. Hence, while CCC may fail in the four-dimensional manifold of general relativity, it may well be restored due to the extra physical effects arising from our transition itself to a higher-dimensional spacetime continuum. Such considerations would inspire a study of gravitational collapse in higher dimensions. From such a perspective, many papers have reported results in recent years on spherically symmetric collapse of dust (where the pressures are identically taken to be zero) in higher dimensions [8-11]. The recent revival of interest in this problem is motivated to an extent by the Randall-Sundrum brane-world scenario [12]. Various authors considered specialized and different subcases of the general problem of dust collapse in higher spacetime dimensions. For example, the marginally bound case in a general spacetime dimension was studied in [9], whereas the same case was studied in [10], however, with an additional and physically motivated restriction on the initial density profiles, namely that the first derivative of the initial density distribution for the collapsing cloud must be vanishing. Also, the non-marginally bound case, however, with the geometric assumption that spacetime is self-similar, was examined in [11] for a five-dimensional model.

Such studies do provide us with an idea of what is possible in gravitational collapse as far as its end state spectrum is concerned. It is obvious from the work in this area so far that any possible proof of CCC must be

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inspired by such additional physical inputs (one of these being a possible transition to higher dimension) into our current framework of thinking. Any such alternatives would be worth exploring due to the fundamental significance of CCC in black hole physics. The point is, if naked singularities did indeed develop in realistic gravitational collapse of massive objects, they may have properties which would be rather different from those of black holes both theoretically as well as observationally, and a comparison of these two cases may prove to be quite interesting.

From such a perspective, we investigate here the issue of BH/NS end states in a higher dimensional dust collapse in some detail, in order to bring out in a transparent manner the effect of dimensions on the final fate of the evolution of the matter cloud which collapses from a given regular initial data. A spherically symmetric collapse is considered in $N \geq 4$ dimensions and the matter form is chosen to be dust as this is a well-studied case. In fact, there have been suggestions in the past that at the vicinity of the collapse, the in-falling velocity of the matter shells could be so high that the effects of pressures are negligible and hence dust may be a good approximation [13]. Apart from that, dust collapse is worth investigation as it has continued to serve as a basic paradigm in black hole physics.

As compared to some of the studies of dust collapse in higher dimensions mentioned above, we deal here within a general framework, i.e. there is no restriction adopted on dimensions, the spacetime is not assumed to be self-similar, which is a somewhat restrictive condition for collapse models, and we treat marginally and non-marginally bound cases together by means of a unified treatment. Also, we make no assumptions on the nature of the initial data, i.e. the initial density and velocity profiles apart from their being regular, and these are taken to be suitably differentiable functions of the comoving radial coordinate at the origin. Our results thus generalize earlier results on the topic and provide a unified treatment. An additional advantage that is derived due to the treatment here is that the dependence of the BH/NS end states on the nature of initial data from which the collapse evolves, is brought out rather clearly. The methodology used also allows us to consider in some detail the genericity and stability aspects related to the occurrence of naked singularities in gravitational collapse.

To predict the final state of collapse for a given initial mass and velocity distribution, we study here the singularity curve resulting from the collapse of successive matter shells, the tangent to which at the singularity is related to the radially outgoing null geodesic equation. Hence, by determining the sign of the tangent of the singularity curve at the central singularity we can integrate the null geodesics close to the singularity, and find whether a future directed radially outgoing null geodesic comes out from the shell-focusing central singularity. This allows us to determine the final fate of collapse in terms of either a black hole, or a naked singularity. It turns out that for CCC to be true *even in higher dimensions*, one must impose a number of constraints for the given system. The physical relevance of such assumptions is discussed here in some detail.

The paper is organized as follows. In the next section we set up the Einstein equations, and the regularity conditions for collapse are specified. In Section III, continually collapsing matter clouds are considered and it is shown how the initial data in terms of the initial distributions of the density and velocity profiles determine the final fate of the collapse in terms of either a black hole or a naked singularity. The dependence of these BH/NS phases on the nature of initial data is brought out clearly here. In Section IV we discuss cosmic censorship in higher dimensions from the perspective of our results, and the genericity and stability aspects related to naked singularity formation are also considered in some detail. The case of marginally bound collapse is discussed in some detail here together with some other cases of interest, relating this to the question of when it may be possible to restore the cosmic censorship for such models in terms of certain possible physical assumptions for the initial density profiles. Some conclusions and remarks are summarized in the final Section V.

II. EINSTEIN EQUATIONS AND REGULARITY CONDITIONS

To study the collapse of a spherical dust cloud, we choose a general spherically symmetric co-moving metric in $N \geq 4$ dimensions which has the form,

$$ds^2 = -dt^2 + e^{2\psi(t,r)}dr^2 + R^2(t,r)d\Omega^2_{N-2} \tag{1}$$

where,

$$d\Omega^2_{N-2} = \sum_{i=1}^{N-2}\left[\prod_{j=1}^{i-1}\sin^2(\theta^j)\right](d\theta^i)^2 \tag{2}$$

is the metric on $(N-2)$ sphere. The energy-momentum tensor of the dust has the form,

$$T^t_t = \rho(t,r);\;\; T^r_r = 0;\;\; T^{\theta^i}_{\theta^i} = 0 \tag{3}$$

We take the matter field to satisfy the *weak energy condition*, i.e. the energy density measured by any local observer be non-negative, and so for any timelike vector V^i, we must have,

$$T_{ik}V^iV^k \geq 0 \implies \rho \geq 0 \tag{4}$$

In the case of a collapsing cloud, the cloud has a finite boundary, $0 < r < r_b$, outside which it is matched to a Schwarzschild exterior. The range of the coordinates for the metric is then $0 < r < \infty$, and $-\infty < t < t_s(r)$ where $t_s(r)$ corresponds to the singular epoch $R = 0$.

Using the above conditions, we can write the N-dimensional Einstein equations as [14],

$$(N-2)\left\{\frac{(N-3)}{2R^2}(1+\dot{R}^2-R^{'2}e^{-2\psi})+\frac{1}{R}(\dot{\psi}\dot{R}+e^{-2\psi}\psi'R'-e^{-2\psi}R'')\right\} = \rho \quad (5)$$

$$(N-2)\left\{\frac{N-3}{2R^2}(R'2e^{-2\psi}-\dot{R}^2-1)+\frac{\ddot{R}}{R}\right\}=0 \quad (6)$$

$$\frac{(N-2)}{R}\left\{2(\dot{R}'-\dot{\psi}R')\right\}=0 \quad (7)$$

$$^{N}G^{i}_{i} \ \ (i=2,\cdots,N-2)=0 \quad (8)$$

where $^{N}G^{k}_{l}$ is the N-dimensional Einstein tensor. Integrating the first part of equation (7) we get,

$$e^{2\psi}=\frac{R^{'2}}{1+f(r)} \quad (9)$$

where $f(r)$ is an arbitrary function of co-ordinate r, and $f(r)>-1$. Hence the generalized Tolman-Bondi-Lemaitre (TBL) metric in N-dimensions becomes,

$$ds^2=dt^2-\frac{R^{'2}}{1+f(r)}dr^2-R^2(t,r)d\Omega^2_{N-2} \quad (10)$$

Now substituting equation(9), in (5) and (6), we have,

$$\frac{(N-2)U'}{2R^{(N-2)}R'}=\rho, \quad \frac{(N-2)\dot{U}}{2R^{(N-2)}\dot{R}}=0 \quad (11)$$

where we define and get by solving equation(11),

$$U=R^{(N-3)}(\dot{R}^2-f(r)) \ \ ; \ \ U=F(r) \quad (12)$$

Here $F(r)$ is another arbitrary function of the comoving coordinate r. In spherically symmetric spacetimes $F(r)$ is the *mass function* which describes the mass distribution of the dust cloud and $f(r)$ is the *energy function* for the collapsing shells. Thus using equations (11) and (12), we finally get the required equations of motion as,

$$\frac{(N-2)F'}{2R^{(N-2)}R'}=\rho \ \ ; \ \ \dot{R}^2=\frac{F(r)}{R^{(N-3)}}+f(r) \quad (13)$$

As seen from the above, once the mass function is specified at the initial epoch the energy function $f(r)$ fully specifies the velocity distribution of infalling shells. Also, the energy condition then implies $F'\geq 0$. It follows from the above that there is a spacetime singularity at $R=0$ and at $R'=0$. The latter are called 'shell-crossing singularities' which occur when successive shells of matter cross each other. These have not been considered generally to be genuine spacetime singularities, and possible extensions of spacetime have been investigated through the same [15]. On the other hand, the singularity at $R=0$ is where all matter shells collapse to a zero physical radius, and hence has been known as a 'shell-focusing singularity'. The nature of this singularity has been investigated extensively in four-dimensional spacetimes (see e.g. references in [4], and other reviews), and it is known for the case of spherical dust collapse that it can be both naked or covered depending on the nature of models one is considering.

The collapsing matter cloud condition implies that $\dot{R}<0$. For dust clouds it follows form the equations of motion that once $\dot{R}<0$ at the initial epoch from where the collapse commences, then at all epochs we have the same condition holding, and thus there is an endless collapse till the shell-focusing singularity $R=0$ is reached. In other words, there is no bounce possible in dust collapse once the collapse has initiated with $\dot{R}<0$. We use the scaling independence of the comoving coordinate r to write (see e.g. [4]),

$$R(t,r)=rv(t,r) \quad (14)$$

where,

$$v(t_i,r)=1 \ \ ; \ \ v(t_s(r),r)=0 \ \ ; \ \ \dot{v}<0 \quad (15)$$

This means we have scaled the coordinate r in such a way that at the initial epoch $R=r$ and at the singularity, $R=0$. It should be noted that we have $R=0$ both at the regular center $r=0$ of the cloud, and at the spacetime singularity where all matter shells collapse to a zero physical radius. The regular center is then distinguished from the singularity by a suitable behaviour of the mass function $F(r)$ so that the density remains finite and regular there at all times till the singular epoch. The introduction of the parameter v then allows us to distinguish the spacetime singularity from the regular center, with $v=1$ all through the initial epoch, including the center $r=0$, which then decreases monotonically with time as collapse progresses to value $v=0$ at the singularity $R=0$.

From the equations of motion it is evident that to have a regular solution over all space at the initial epoch, the two free functions $F(r)$ and $f(r)$ must have the following forms,

$$F(r)=r^{(N-1)}\mathcal{M}(r); \ \ f(r)=r^2b(r) \quad (16)$$

where $\mathcal{M}(r)$ and $b(r)$ are at least C^1 functions of r for $r=0$, and at least C^2 functions for $r>0$. This is dictated by the condition that the density and energy distributions must be regular at the initial epoch and should not be blowing up. This is because if the mass function F did not go as power at least r^{N-1} closer to the origin, then as implied from the equations of motion, the density will be singular at the origin $r=0$ as it will diverge there,

and that cannot be accepted as regular initial data for collapse. Similarly, equation (12) implies that $f(r)$ is determined once the velocity profile is specified, and vice-versa, for a given initial density distribution. Since the center of the cloud is taken to be at rest in any spherically symmetric distribution, the leading term in the energy profile must be at least r or higher. Then again equation (12) implies the behaviour for $f(r)$ as above.

III. CONTINUAL COLLAPSE AND BH/NS END STATES

We now consider the endless collapse of a dust cloud to a final shell-focusing singularity at $R = 0$, where all matter shells collapse to a zero physical radius. In particular, we analyze specifically the nature of the central singularity at $R = 0, r = 0$ in detail to determine when it will be covered by the event horizon, and when visible and causally connected to outside observers. If there are future directed families of non-spacelike curves, coming out from the singularity and reaching faraway observers, then the singularity will be naked. The absence of such families will give a covered case when the result is a black hole.

In the following, a higher dimensional collapse as discussed above is considered and we shall show how the initial density and energy distribution prescribed at the initial epoch from which the collapse commences, completely determine if there will be families of non-spacelike trajectories coming out of the singularity. We shall show that for a generic situation, given an initial density distribution for the collapse to develop, it is always possible to choose an energy profile so that the collapse of this density profile ends in a naked singularity, or one could also choose another class of energy distribution so that the same density distribution will end up collapsing so as to create a black hole. That is, given an initial density profile, the outcome in terms of either a black hole or a naked singularity as end states really depends on the class of the energy distribution chosen. The converse will also be seen to hold true, that is, given the initial energy function, one will be able to choose classes of density profiles, subject to the the weak energy condition, so as to give rise to either of the BH/NS end states depending on the choice made. Our results reduce to the usual dust collapse models of general relativity when $N = 4$.

With the regular initial conditions as above, equation(13) can be written as,

$$v^{\frac{N-3}{2}}\dot{v} = -\sqrt{\mathcal{M}(r) + v^{(N-3)}b(r)} \qquad (17)$$

Here the negative sign implies that $\dot{v} < 0$, that is, the matter cloud is collapsing. Integrating the above equation with respect to v, we get,

$$t(v,r) = \int_v^1 \frac{v^{\frac{N-3}{2}}dv}{\sqrt{\mathcal{M}(r) + v^{(N-3)}b(r)}} \qquad (18)$$

We note that the co-ordinate r is to be treated as a constant in the above equation. Expanding $t(v,r)$ around the center, we get,

$$t(v,r) = t(v,0) + r\mathcal{X}(v) + r^2\frac{\mathcal{X}_2(v)}{2} + r^3\frac{\mathcal{X}_3(v)}{6} + \cdots \qquad (19)$$

where the function $\mathcal{X}(v)$ is given by,

$$\mathcal{X}(v) = -\frac{1}{2}\int_v^1 \frac{v^{\frac{N-3}{2}}(\mathcal{M}_1 + v^{(N-3)}b_1)dv}{(\mathcal{M}_0 + v^{(N-3)}b_0)^{\frac{3}{2}}} \qquad (20)$$

where,

$$b_0 = b(0) \;\; ; \;\; \mathcal{M}_0 = \mathcal{M}(0) \;\; ; \;\; b_1 = b'(0) \;\; ; \;\; \mathcal{M}_1 = \mathcal{M}'(0) \qquad (21)$$

Thus, the time taken for the central shell to reach the singularity is given as

$$t_{s_0} = \int_0^1 \frac{v^{\frac{N-3}{2}}dv}{\sqrt{\mathcal{M}_0 + v^{(N-3)}b_0}} \qquad (22)$$

From the above equation it is clear that for t_{s_0} to be defined,

$$\mathcal{M}_0 + v^{(N-3)}b_0 > 0 \qquad (23)$$

In other words, the continual collapse condition implies the positivity of the above term. Hence the time taken for other shells to reach the singularity can be given by the expansion,

$$t_s(r) = t_{s_0} + r\mathcal{X}(0) + \mathcal{O}(r^2) \qquad (24)$$

Also, from equation(17) and (19) we get for small values of r, along constant v surfaces,

$$v^{\frac{N-3}{2}}v' = \sqrt{(\mathcal{M}_0 + v^{(N-3)}b_0)}\,(\mathcal{X}(v) + r\mathcal{X}_2(v) + \cdots) \qquad (25)$$

Now we can easily see that the value of $\mathcal{X}(0)$ depends on and is completely characterized by the functions $M(r)$ and $b(r)$, which in turn specify fully the initial mass and energy distributions for the collapsing matter. Specifying these functions is equivalent to specifying the regular initial data for collapse on the initial surface $t = 0$. In other words, a given set of density and energy distribution completely determines the slope to the singularity curve at the origin, which is the central singularity. Also, it is evident that given any one of these two profiles we can always choose the other one in such a manner so that the quantity $\mathcal{X}(0)$ will be either positive or negative.

In order to determine the visibility or otherwise of the central singularity, we now need to analyze the behaviour of non-spacelike curves in the vicinity of the singularity and the causal structure of the trapped surfaces.

The boundary of the trapped surface region of the space-time is given by the apparent horizon within the collapsing cloud, which is given by the equation,

$$\frac{F}{R^{N-3}} = 1 \qquad (26)$$

What we need to determine now is when there will be families of non-spacelike paths coming out of the singularity, reaching far away observers, and when there will be none. The visibility or other wise of the singularity is decided accordingly. Broadly, it can be stated that if the neighborhood of the center gets trapped earlier than the singularity, then it is covered, otherwise it is naked with families of non-spacelike future directed trajectories escaping away from it. By determining the nature of the singularity curve and its relation to the initial data, we are able to deduce whether the trapped surface formation in collapse takes place before or after the singularity. It is this causal structure that determines the possible emergence or otherwise of non-spacelike paths from the singularity, and settles the final outcome in terms of either a BH or NS.

To consider the possibility of existence of such families, and to examine the nature of the singularity occurring at $R = 0$, $r = 0$ for the scenario under consideration, consider the outgoing radial null geodesics equation,

$$\frac{dt}{dr} = e^{\psi} \tag{27}$$

The singularity occurs at a point $v(t_s(r), r) = 0$, which corresponds to $R(t_s(r), r) = 0$. Therefore, if we have any future directed null geodesics terminating in the past at the singularity, we must have $R \to 0$ as $t \to t_s$. Now writing equation(27) in terms of variables $(u = r^{\alpha}, R)$ where $\alpha > 1$, we have,

$$\frac{dR}{du} = \frac{1}{\alpha} r^{-(\alpha-1)} R' \left[1 + \frac{\dot{R}}{R'} e^{\psi}\right] \tag{28}$$

Choosing $\alpha = \frac{N+1}{N-1}$, and using equation(13) together with the collapse condition $\dot{R} < 0$, we get,

$$\frac{dR}{du} = \frac{N-1}{N+1}\left(\frac{R}{u} + \frac{v' v^{\frac{N-3}{2}}}{(\frac{R}{u})^{\frac{N-3}{2}}}\right)\left(\frac{1 - \frac{F}{R^{N-3}}}{e^{-\psi} R' \left(e^{-\psi} R' + \left|\dot{R}\right|\right)}\right) \tag{29}$$

If null geodesics terminate at the singularity in the past with a definite tangent, then at the singularity we have $\frac{dR}{du} > 0$, in the (u, R) plane which must have a finite value.

In the case under consideration, all singularities for $r > 0$ are covered since $\frac{F}{R} \to \infty$ in the limit of approach to the singularity in that case, and hence $\frac{dR}{du} \to -\infty$. Therefore only the singularity at the central shell could be naked.

In order to see possible emergence of null geodesics from the central singularity, we now need to analyze equation(29). Let us calculate the limits of the concerned functions in equation(29) at the central singularity. From equation(9) we get that in the limit of $t \to t_s, r \to 0$ we have $e^{-\psi} R' \to 1$. Also from equation(17) and (25), we have in this limit $\dot{R} \to 0$. It then follows in general, from the Einstein equations discussed above, that the term F/R^{N-3} goes to zero in this limit.

We would like to find when there will be future directed null geodesics coming out from the central singularity with a well-defined and definite positive tangent in the (t, r) or (R, u) plane, thus making the singularity visible. Let us define the tangent to the null geodesic at the singularity as,

$$x_0 = \lim_{t \to t_s} \lim_{r \to 0} \frac{R}{u} = \left.\frac{dR}{du}\right|_{t \to t_s; r \to 0} \tag{30}$$

Using equations (29) and (25), along with the required limits as above, we get,

$$x_0^{\frac{N-1}{2}} = \frac{N-1}{2} \sqrt{\mathcal{M}_0} \mathcal{X}(0) \tag{31}$$

Let us now deduce the necessary and sufficient conditions for a naked singularity to exist, that is, for null geodesics with a well-defined tangent to come out from the central singularity. Suppose we have $\mathcal{X}(0) > 0$, then we always have (from equation(31)), $x_0 > 0$ and then in the (R, u) plane, the equation for the null geodesic that comes out from the singularity is given by

$$R = x_0 u \tag{32}$$

In other words, equation(32) is a solution of the null geodesic equation in the limit of the central singularity. Thus given $\mathcal{X}(0) > 0$, we can always construct a solution of radially outgoing null geodesics emerging from the singularity. This makes the central singularity visible. In the (t, r) plane, the null geodesics above near the singularity will be given as,

$$t - t_s(0) = x_0 r^{\frac{N+1}{N-1}} \tag{33}$$

It follows that $\mathcal{X}(0) > 0$ implies $x_0 > 0$ and we get radially outgoing null geodesics emerging from the singularity, giving rise to the central naked singularity.

On the other hand, if $\mathcal{X}(0) < 0$, then we see that the singularity curve is a decreasing function of r. Hence the region around the center gets singular before the central shell, and after that it is no more in the spacetime. Now if there would have been any *outgoing* null geodesic from the central singularity, it must then go to a singular region or outside the spacetime, which is impossible. Hence when $\mathcal{X}(0) < 0$, we always have a black hole solution.

If $\mathcal{X}(0) = 0$ then we will have to take into account the next higher order non-zero term in the singularity curve equation, and do a similar analysis by choosing a different value of α in equation (28).

We have thus shown above that $\mathcal{X}(0) > 0$ is the necessary and sufficient condition for null geodesics to come out from the central singularity with a definite positive tangent. It should be noted, however, that in general the dependence of R on r along the outgoing null geodesics from the singularity does not necessarily have to be of a power-law form. However, in order to satisfy the regularity and physical relevance, examining the trajectories which come out with a regular and well-defined tangent

is physically more appealing, which is the case we have examined here.

We show below for completeness, however, that if null geodesics of any form come out at all, then those with definite tangent also must emerge from the central singularity. Towards this, let us consider the equation of apparent horizon, which is from equations (26) and (18) given by,

$$t_{ah}(r) = t_{s_0} + r\mathcal{X}(0) + r^2\frac{\mathcal{X}_2(0)}{2} + \cdots - \mathcal{O}(r^{\frac{N-1}{N-3}}) \quad (34)$$

Since the apparent horizon is a well-behaved surface as one initiates close to $r = 0$, for a spherical dust collapse, hence we can say that the singularity curve for the collapse and the derivatives around the center are also well-defined, as the same coefficients are present in both (34) and (19). Also, this shows that whenever $\mathcal{X}(0)$ is negative, the region around the center gets trapped before the central singularity, giving a sufficient condition for a black hole to develop. It follows that if null geodesics are coming out, then at least one of the coefficients $\mathcal{X}$ must be non-vanishing and positive. Then, as we have already shown, null geodesics with definite tangent will come out from the central singularity.

From the above it follows that *in the absence of a null geodesic with a definite tangent there cannot be any null geodesics coming out of the singularity.*

It is also clear now, from equation (20), that whether $\mathcal{X}(0) > 0$ or otherwise is fully determined by the regular initial data for collapse, in terms of the given initial density and energy distribution for the collapsing shells. It thus follows that the initial data here completely determines the final fate of collapse in terms of BH/NS end states. We shall discuss this further in the next section.

IV. COSMIC CENSORSHIP AND GENERICITY ISSUES

It is now possible to examine the question of validity of cosmic censorship in a higher dimensional collapse scenario under consideration. Since we can explicitly find out as pointed out above, when families of non-spacelike geodesics can come out of the singularity or otherwise, we can address now this question below in some detail, and we also discuss some related issues.

To focus the discussion, let us consider a model initial density profile as given by,

$$\rho(t_i, r) = \rho_0 + r\rho_1 + r^2\frac{\rho_2}{2!} + r^3\frac{\rho_3}{3!} + \cdots \quad (35)$$

and we write the function $\mathcal{M}(r)$ as,

$$\mathcal{M}(r) = \sum_{n=0}^{\infty}\mathcal{M}_n r^n \quad ; \quad \mathcal{M}_n = \frac{2\rho_n}{(N-2)(N+n-1)n!} \quad (36)$$

along with an energy profile as specified by,

$$b(r) = b_0 + rb_1 + r^2\frac{b_2}{2!} + \cdots \quad (37)$$

A. Marginally bound collapse

In the first place, let us consider the class of *marginally bound* collapse models for a more transparent understanding of the problem. This is the case when the energy function $b(r)$ above vanishes identically for the collapsing shells. In such a situation, the first non-vanishing coefficient $\mathcal{X}_n(0)$,where $n > 0$, as described in equation (19) are given by,

$$\mathcal{X}_n(0) = -\frac{n!}{N-1}\left(\frac{\mathcal{M}_n}{\mathcal{M}_0^{\frac{3}{2}}}\right) \quad (38)$$

The quantities $\mathcal{M}_n$ are described in equation (36). Now it is evident that whenever $\rho_1 < 0$, we will get a naked singularity *in all dimensions*, whereas $\rho_1 > 0$ always results in a black hole. The case $\rho_1 < 0$ corresponds to the physical situation when the density decreases with increasing comoving radius r, as one would typically expect the density to be highest at the center and then gradually decrease as we move out in any realistic configuration such as a massive star. Further, we note that the above conclusion is not dependent on the magnitude of ρ_1, but only on its sign, that is density must decrease away from the center with the density gradient being non-zero. Thus it becomes clear that it is the *density inhomogeneity* that delays the formation the trapped surfaces, thus causing a naked singularity. This is closely connected to the non-vanishing *spacetime shear*, and for a discussion of how shear distorts the geometry of trapped surfaces closer to the spacetime singularity, we refer to [16].

Let us, however, assume that the initial density distribution has all odd terms in r vanishing, i.e. it admits no 'cusps' at the center and that it is either sufficiently differentiable, or is a smooth and analytic function of r. In that case, we must have $\rho_1 = 0$. Then we get from equation (25) that in the neighborhood of the singularity, the behavior of v is given by,

$$\lim_{t\to t_s}\lim_{r\to 0} v = \left[\frac{N-1}{4}\sqrt{\mathcal{M}_0}\mathcal{X}_2(0)\right]^{\frac{2}{N-1}} r^{\frac{4}{N-1}} \quad (39)$$

Also, in the same limit the function $\frac{F}{R^{N-3}}$ has the form

$$\lim_{t\to t_s}\lim_{r\to 0}\frac{F}{R^{N-3}} = \frac{r^2\mathcal{M}_0}{v^{(N-3)}} \quad (40)$$

Thus it is clear from equations (39) and (40), that if $N > 5$ then for $\lim_{t\to t_s}, \lim_{r\to 0}$, $\frac{F}{R} \to \infty$ and thus the end state of collapse will always be a black hole (see also [10]). It thus follows that for a marginally bound dust collapse, with $\rho_1 = 0$, i.e. when the initial density profile is sufficiently differentiable and smooth, the CCC is always respected in a higher dimensional spacetime with $N = 6$ or higher.

B. Critical value of ρ_2 in five-dimensional case

We note that in the conclusion above, the spacetime dimension has to be either six, or higher. Let us consider the case when the spacetime dimension is five, but still with an analytic initial density profile.

In case of a five-dimensional marginally bound collapse with $\rho_1 = 0$, we can write the tangent to the outgoing radial null geodesic at the singularity in the (R,u) plane as,

$$x_0^2 = \sqrt{\mathcal{M}_0}\mathcal{X}_2(0)\frac{\left[1-\sqrt{\frac{F}{R^2}}\right]}{\left[1+\sqrt{\frac{F}{R^2}}\right]} \tag{41}$$

The sufficient condition for the existence of an outgoing null geodesic from the singularity is we must have $x_0 > 0$, which in the above case amounts to,

$$\xi \equiv \frac{\mathcal{M}_2}{\mathcal{M}_0^2} < -2 \tag{42}$$

But again, the outgoing null geodesic should be within the spacetime *i.e* the slope of the geodesic must be less than that of the singularity curve,

$$\lim_{t\to t_s}\lim_{r\to 0}\left(\frac{dt}{dr}\right)_{null} \leq \left(\frac{dt}{dr}\right)_{sing} \tag{43}$$

From the above equation we get the condition,

$$\xi = \frac{\mathcal{M}_2}{\mathcal{M}_0^2} \leq -8 \tag{44}$$

Thus from equation (42) and (44) we see that for an outgoing null geodesic from the singularity to exist we must have $\xi \leq \xi_c = -8$, in which case we get a naked singularity, and otherwise a black hole results as collapse end state.

We note that this situation has an interesting parallel to the four-dimensional collapse scenario, where we have a similar critical value existing, however, it is for the coefficient ρ_3, when both ρ_1 and ρ_2 are vanishing [17]. Thus, with the increase of the spacetime dimension by one, the criticality separating the BH/NS phases shifts at the level of second density derivative from the earlier third density derivative.

An interesting observation we could make here is that for $\xi < -2$ we have an increasing apparent horizon at the singularity. The apparent horizon is given by $R = F$ so it initiates at the central singularity $r = R = 0$, and in the above case it is increasing in time (as opposed to the Oppenheimer-Snyder case of homogeneous dust collapse). Thus for the range $-2 > \xi > -8$ no trapped surface is formed till the singularity epoch, but still we get a black hole as the collapse end state. This confirms that *the absence of a trapped surface till the singularity is necessary, but not a sufficient condition for the formation of a naked singularity.*

C. Marginally bound collapse with initial homogeneous density

It is useful to note here that the well-known Oppenheimer-Snyder class of collapse solutions is a special case of marginally bound dust collapse in four dimensions in which the initial density profile is homogeneous, that is, $\mathcal{M}_n(n > 0) = 0$ for all n. The point is, if the initial density is homogeneous but if the collapse is *not* marginally bound, then the non-zero energy function f could inhomogenize the collapse at later epochs. In the present case, as $f = 0$, at all later epochs also the density remains a function of time only, that is, it is homogeneous at all later times as well. Thus we clearly see that the final outcome of this class of collapse is always a black hole. Furthermore from equations (24) and (19) we see that,

$$t_s(r) = t_{s_0} \quad ; \quad v(t,r) = v(t) \tag{45}$$

As the scale function v is independent of r, all the shells collapse simultaneously to the singularity. The time taken to reach the singularity is given as,

$$t_{s_0} = \frac{2}{(N-1)\mathcal{M}_0^{\frac{1}{2}}} \tag{46}$$

Thus, as we go to higher dimensions, for a given density, the time taken to reach the singularity will reduce.

It is, however, interesting to note that even if we start with an initial homogeneous density profile but allow for non-zero initial radial and tangential pressures of the form,

$$p_r(t,r) = 1 \;\; ; p_{\theta_0}(r) = 1 + p_{\theta_2}r^2 + p_{\theta_3}r^3 + \cdots \tag{47}$$

then as shown in [18], we have

$$\mathcal{X}(0) = -\frac{1}{3}\int_0^1 \frac{v^{\frac{N+5}{2}}(p_{\theta_3})}{v^{(N-1)}(p_{\theta_2}-\frac{1}{3})+\frac{2}{3}} \tag{48}$$

Thus it is seen that a negative p_{θ_3} coefficient does lead to a naked singularity.

This is similar to the case of a collapse which is not marginally bound, where an initially homogeneous density profile can turn inhomogeneous at later epochs due to the non-vanishing shell velocities. In the same way, in the case above also the non-vanishing pressures could inhomogenize the initially homogeneous density distribution at later epochs to cause a naked singularity eventually. It has to be noted, all the same, that the equation of state in the situations such as above could be considered to be some what peculiar (though matter is fully normal, satisfying the positivity of energy condition, and collapse conditions are fully regular). To state this differently, one can argue that the models where only purely tangential pressures are taken to be non-vanishing may not be considered to be physically realistic, and if we choose the equation of state to be say $p = k\rho, k > 0$, or any homogeneous equation of state, then when the initial density

profile is taken to be homogeneous, then so will be the initial pressures, and then the collapse will end up in a black hole only, and no naked singularity will arise.

D. Genericity and stability of naked singularities

In the above, we have discussed various special subcases of a higher dimensional collapse scenario which result either in a black hole or a naked singularity end state, depending on values and behaviour of the parameters involved, and we also discussed the possible preservation of CCC under various set of assumptions.

It is necessary however, to look at the situation in a collective manner if we are to gain any insight on the genericity and stability aspects connected to the naked singularities forming in gravitational collapse. It is well-known of course, that the genericity and stability are quite involved issues in the general theory of relativity, and that there does not exist any well-defined way to test the same in a unique manner for a given situation. Also, there may be different kinds of stabilities involved. For example, we can ask here, if the conclusions above will be stable to non-spherical perturbations, or when forms of matter more general than dust are considered and so on. Such issues are worth a separate and detailed investigation, and will be crucial towards the important problem of collapse end states.

At a somewhat different, but still quite interesting level, we can inquire about the stability of BH/NS end states with respect to the perturbations in the initial data space which determines the final outcome of collapse as we have seen above. As pointed out here, this is a function space consisting of all possible mass functions F and energy functions f. It is worth knowing how, for example, a naked singularity end state would be affected when one moves from a given density and energy profile (which gave rise to this state) to a nearby density or energy profile in this space of all initial data.

The issue of how given density and energy distributions determine the final collapse end state has been discussed quite extensively in the usual four-dimensional dust collapse models, though using a somewhat different methodology [17]. These results were completed to give a full and general treatment of four-dimensional case in [19], and the typical result is that given any density profile, one could choose the energy profile (and vice-versa), so that the collapse end state would be either a black hole, or a naked singularity depending on this choice.

Our results here generalize this to the case of a higher dimensional collapse situation, and our method now allows us to make a more definite statement on genericity of naked singularity formation (in the sense stated above). As we see from equation (20), the quantity $\mathcal{X}(0)$ is fully determined from the initial data functions and their first derivatives. Once it is positive, the collapse ends in a naked singularity and a negative value gives black hole. It follows by continuity that given a density profile, if the energy profile chosen is such that the collapse ends in NS, i.e. $\mathcal{X}(0) > 0$, then there is a whole family of near by velocities such that this will continue to be the case, and the NS will form an open subspace in the initial data space. Same of course holds for BH formations as well, and both these are neatly separated open regions in the initial data space. But if we take on physical grounds both ρ_1 and b_1 to be vanishing, then from equation (20) we have $\mathcal{X}(0) = 0$, and CCC may be restored.

V. CONCLUDING REMARKS

In this final section, we give several concluding remarks and observations.

1. It follows that if we can suitably motivate physically all the assumptions such as those discussed above, then it may be possible to restore CCC in a higher dimensional spacetime. Let us discuss such conditions in some detail. That the equation of state must be dust-like in the final phases of collapse is a strong assumption, but it is not a possibility that can be completely ruled out (see e.g. [13]). After all, we know very little on the equations of state, especially how it would be like in the advanced stages of collapse. Also, it is quite possible that in the very late stages of collapse the configuration is very much like a marginally bound one in the vicinity of the singularity. The introduction of pressures may or may not change such a scenario.

All the same, in our view, the assumption $\rho_1 = 0$ is a tricky one, and this has been extensively discussed in the past within the context of collapse in four-dimensions. While it may be quite convenient to deal with smooth and analytic density profiles, especially when it comes to numerical models, it should not be forgotten that after all this is only an extra assumption, and that the basic equations of general relativity do not demand any such constraint. Neither it is clear astrophysically that the interiors of the stars must necessarily have analytic density or energy distribution. In certain equilibrium cases, the field equations imply that these have to be smooth, but this need not be true in general, and especially the dynamically developing collapse situations could be quite different.

We thus conclude that each of the above assumptions require further scrutiny and sufficient physical motivation so as to arrive at any definite conclusion on the status of CCC in a higher dimensional spacetime. However, this will be certainly worth the effort, given the importance of these issues in black hole physics.

2. Let us consider the scenarios when some of the above assumptions break down. We have already noted earlier that when ρ_1 is non-vanishing, then the collapse ends in a naked singularity in all dimensions, including $N = 4$. Again, our considerations above immediately imply that whenever spacetime is *not* marginally bound, the collapse always results into both the BH/NS phases as collapse end states, depending on the nature of ini-

tial data, *irrespective of ρ_1 being either zero or non-zero.* That is, in a more generic non-marginally bound case, the condition $\rho_1 = 0$ does not save the CCC. However, if we take on physical grounds both $b_1 = \rho_1 = 0$ then from equation (20) $\mathcal{X}(0) = 0$ and CCC could be preserved.

3. Consider the situation when we must believe some how that in the later stages of collapse the form of matter cannot be dust-like, and that non-dust forms of matter, and effects of pressures must be suitably taken into account. In such a case, as pointed out above, it is seen that even if one considered only homogeneous initial density profiles (however, with non-zero initial pressures), then also the pressure by it-self can cause sufficient distortions in the formation of the apparent horizon so as to cause a naked singularity as end state of collapse, rather than a black hole. However, if the equation of state is homogeneous, together with initial data being homogeneous density profile, then no naked singularity will appear.

4. We would like to suggest that there may be some hope, as outlined above, to recover CCC while we transit to a higher dimensional spacetime arena. This is subject to validity of several extra physical inputs as we described above. On the other hand, once we move to more general situations of either a non-marginally bound case, or with a more general form of matter, or without restrictive extra-assumptions on the nature of the initial density profiles, then generically both the BH/NS phases could result as end states of collapse in a higher dimensional spacetime scenario. It would be fair to state that dynamical collapse in general relativity offers a rich spectrum of possibilities to investigate.

5. Finally, we point out that the formalism given here brings out the role of initial data in causing BH/NS end states for the four-dimensional dust collapse also in a clear and transparent manner, completing earlier results in this direction. To be specific, it is seen, using equation (20), that given any initial density distribution for the cloud, one can always choose a suitable energy profile so that the evolution could end in either of a black hole or a naked singularity depending on the choice made. In other words, there is a non-zero measure of energy distributions which will take the given density profile to a black hole, and the same holds for a naked singularity to evolve from the same initial density. The converse is also true, namely, given any initial energy distribution, one can choose the density profiles which give rise to either of these end states.

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